**Enrollment No:** _______________________ **Exam Seat No:**_______________________

# C. U. SHAH UNIVERSITY

## Summer Examination-2022

**Subject Name : Metric Space**

**Subject Code: 4SC05MES1** **Branch: B.Sc. (Mathematics)**

**Semester: 5** **Date: 26/04/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

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**Q-1 Attempt the following questions:** **[14]**

**a)** Let $(X, d)$ be a metric space and $E \subset X$.Then set $E$ is said to be dense set if … **(01)**

1) $E' = X$
2) $E' = E$
3) $\bar{E} = X$
4) $\bar{E} = E$

**b)** Which of the following subset of $\boldsymbol{R}$is not closed? **(01)**

1) $\{1,2, \dots ,10\}$
2) $\{1,2,3, \dots\}$
3) $[0,100]$
4) $(-1,6]$

**c)** If $E = [1,3]$ is subset of metric space $\boldsymbol{R}$then $E^{\circ} =$ _____ **(01)**

1) $(1,3]$
2) $[1,3)$
3) $(1,3)$
4) $[1,3]$

**d)** Define : Compact Set **(01)**

**e)** Define : Interior Point **(01)**

**f)** Check whether the statement is true or false: If $A \subseteq B$ then $A^{\circ} \subseteq B^{\circ}$. **(01)**

**g)** Define : Metric Space **(01)**

**h)** Check whether the statement is true or false: Every closed and bounded subset of the real line is not compact. **(01)**

**i)** Find $A^{\circ}$ for $A = (0,1]$ **(01)**

**j)** Check whether the statement is true or false: Let $A$ be connected subset of metric space $X$ and $B$ be a subset of $X$ such that $A \subseteq B \subseteq \bar{A}$ then $B$ is also connected. **(01)**

**k)** Let $X = \boldsymbol{R}$and $A = \emptyset$ then find $int\ A$ and $ext\ A$. **(02)**

**l)** Define : Continuous function in Metric space **(02)**

**Attempt any four questions from Q-2 to Q-8**

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| **Q-2** | | **Attempt all questions** | **[14]** |
| | **a)** | Prove: i) Finite intersection of open sets of metric space is an open set.<br>ii) Arbitrary intersection of closed sets of metric space is a closed set. | **(06)** |
| | **b)** | Let $(X,d)$ be a metric space and $E \subset X$.If $'a'$ is a limit point of $E$ then show that there are infinitely many points of $E$ in every neighborhood of $'a'$. | **(04)** |
| | **c)** | Define : Closed Set .Show that every finite subset of metric space is closed. | **(04)** |
| **Q-3** | | **Attempt all questions** | **[14]** |
| | **a)** | Let $E_n = (c - \frac{1}{n}, c + \frac{1}{n})$ where $c \in \boldsymbol{N}$is constant and $n \in \boldsymbol{N}$.Compute $\bigcup_{n=1}^{\infty} E_n$and $\bigcap_{n=1}^{\infty} E_n$and determine whether they are open or closed ? | **(06)** |
| | **b)** | Let $X = R$ and define $d: R \times R \to R$ by $d(x,y) = \lvert x - y \rvert$ ,then prove that $(X,d)$ is metric space. | **(05)** |
| | **c)** | Define (i) Derived Set (ii) Dense Set | **(03)** |
| **Q-4** | | **Attempt all questions** | **[14]** |
| | **a)** | Let$(X,d)$be a metric space and $d_1: X \times X \to \boldsymbol{R}$defined by $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$then prove that $d_1$is also a metric on $X$. | **(06)** |
| | **b)** | Show that distinct points of metric space have different neighborhoods. | **(05)** |
| | **c)** | If $(X,d)$ is a metric space and $A, B \subset X$with $A \subset B$ ,then show that$\bar{A} \subset \bar{B}$. | **(03)** |
| **Q-5** | | **Attempt all questions** | **[14]** |
| | **a)** | For a non-empty subset $A$ of metric space $(X,d)$ show that the function $f: X \to \boldsymbol{R}$defined by $f(x) = d(x,A)$ , $x \in X$ is uniformly continuous. Also show that $f(x) = 0$ if and only if $x \in \bar{A}$. | **(07)** |
| | **b)** | Let $(X,d)$ be a complete metric space and $\{F_n\}$ be a decreasing sequence of non-empty closed subsets of $X$ such that $d(F_n) \to 0$ as $n \to \infty$,then show that $F = \bigcap_{n=1}^{\infty} F_n$ contains exactly one point. | **(07)** |
| **Q-6** | | **Attempt all questions** | **[14]** |
| | **a)** | Prove that the derived set of any subset of metric space is a closed set. | **(07)** |
| | **b)** | Let $(X,d_1)$ and $(Y,d_2)$ be any two metric space, then prove that $f: X \to Y$ is continuous if and only if $f^{-1}(G)$ is open in $X$ whenever $G$ is open in $Y$. | **(07)** |
| **Q-7** | | **Attempt all questions** | **[14]** |
| | **a)** | State and prove Banach Fixed Point Theorem. | **(07)** |
| | **b)** | Let $(X,d)$ be a metric space .If $\{x_n\}$ is convergent sequence of points of $X$ then show that $\{x_n\}$ is Cauchy sequence. | **(04)** |
| | **c)** | Show that the sets $A = (5,6)$ and $B = (6,8)$ are separated sets of metric space $\boldsymbol{R}$. | **(03)** |
| **Q-8** | | **Attempt all questions** | **[14]** |
| | **a)** | Define :Cantor Set.Show that Cantor set is a closed set. | **(07)** |
| | **b)** | Show that every compact subset $A$ of metric space $(X,d)$ is bounded. | **(05)** |
| | **c)** | Give an example of subsets $A$ and $B$ of metric space $\boldsymbol{R}$such that $(A \cap B)' \neq A' \cap B'$. | **(02)** |